Confidence Intervals for Proportions

Lecture 32
Sections 9.4

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Outline

Review

- The point estimate is $\hat{p}$.
- The margin of error 1.96 standard deviations.

$$1.96\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}.$$

- The confidence interval centered at $\hat{p}$ is

$$\hat{p} \pm 1.96\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}.$$

95% Confidence Intervals

- The expression

$$\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

  is called the standard error of $\hat{p}$.
- It is denoted $\text{SE}(\hat{p})$.
- So the 95% confidence interval could be written as

$$\hat{p} \pm \text{SE}(\hat{p}).$$

Case Study

Article

Moderate Exercise May Lower Cold Risk

- In the group that did only stretching exercises, 30 out of 62 got colds.
- Use a 95% confidence interval to estimate the true proportion colds among people who do only stretching exercises.
- How should we interpret the confidence interval?

Standard Confidence Levels

- The standard confidence levels are 90%, 95%, 99%, and 99.9%.

Confidence Level	z
90%	1.645
95%	1.960
99%	2.576
99.9%	3.291

The Confidence Interval

- The confidence interval is given by the formula

$$\hat{p} \pm z\sqrt{\frac{\hat{p}(1-\hat{p})}{n}},$$

where z is obtained using the `invNorm` function on the TI-83.

Probability of Error

- We use the symbol α to represent the probability that the confidence interval is in error.
- That is, α is the probability that p is not in the confidence interval.
- In a 95% confidence interval, $\alpha = 0.05$.
- The area in each tail is $\alpha/2$.

Probability of Error

- Let z_A denote the value of z that cuts off a lower tail of area A.
- For example, $z_{0.05}$ is the 5^{th} percentile of Z.
- The confidence interval requires $z_{\alpha/2}$.
- To find $z_{\alpha/2}$, we should compute `invNorm(`$\alpha/2$`)`.
- The confidence interval is

$$\hat{p} \pm z_{\alpha/2}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}.$$

Confidence Level

- Recompute the confidence interval for the incidence of colds among those who do only stretching exercises.
  - 99% confidence interval.
  - 90% confidence interval.
  - 85% confidence interval.
- Which one is widest?
- In which one do we have the most confidence?

TI-83 - Confidence Intervals

- The TI-83 will compute a confidence interval for a population proportion.
  - Press `STAT`.
  - Select `TESTS`.
  - Select `1-PropZInt`. A display appears requesting information.
  - Enter x, the numerator of the sample proportion.
  - Enter n, the sample size.
  - Enter the confidence level, as a decimal.
  - Select `Calculate` and press `ENTER`.

TI-83 - Confidence Intervals

- A display appears with several items.
  - The title `1-PropZInt`.
  - The confidence interval, in interval notation.
  - The sample proportion $\hat{p}$.
  - The sample size.
- How would we find the margin of error?

TI-83 - Confidence Intervals

- Find the 95% confidence interval again for people who do stretching exercises, this time using the TI-83.

Which Confidence Interval is Best?

- All other things being equal, which is better?
  - A large margin of error or a small margin of error?
  - A low level of confidence or a high level of confidence?
  - A smaller sample size or a larger sample size?
- We would like a confidence interval that has a small margin of error and has a high level of confidence associated with it.
- Hey, why not a margin of error of 0 and a confidence level of 100%?

Which Confidence Interval is Best?

- A larger sample size is better only up to the point where its cost is no longer worth its benefit.
- That is, marginal cost vs. marginal benefit.
- That is why we settle for a certain margin of error and a confidence level of less than 100%.
- Statisticians typically use a sample size of around $n = 1000$ and get a margin of error of about 0.03.

Summary

- The general formula for the confidence interval is

$$\hat{p} \pm z_{\alpha/2}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

  where $z_{\alpha/2}$ is obtained from the `invNorm` function on the TI-83.
- The TI-83 function `1-PropZInt` will compute a confidence interval.
- The only way to substantially improve a confidence interval is to take a larger sample.